CHAPTER 6 FACTORING AND APPLICATIONS

Section 6.1 The Greatest Common Factor; Factoring by Grouping

Objective 1 Find the greatest common factor of a list of terms.

Find the greatest common factor for the list of numbers.

1. 12, 18, 30
2. 90, 36, 18
3. 45, 25, 30
4. 14, 28, 56
5. 30, 20, 60
6. 5, 18, 20
7. 12, 18, 24, 36
8. 28, 70, 126, 42
9. 64, 160, 480
10. 84, 28, 308

Find the greatest common factor for the list of terms.

11. $30x,\ 25x^2,\ 15x$
12. $42y^3,\ 28y^2,\ 35y^3$
13. $6ab,\ 18a^2b,\ 26ab^2,\ 38a^2b^2$
14. $10m^3,\ 15m^5,\ 21m^7$
15. $6x^2,\ 9x^4,\ 24x^5$
16. $y^4z^2,\ z^5,\ y^6z^9$
17. $16k^2m^2n^4,\ 32m^3n^5,\ k^4m^8n^7$
18. $49w^5xy,\ w^4x^5y^7,\ 42w^2x^3y^5$
19. $45a^3y^4,\ 75a^3y^2,\ 60a^2y,\ 90a^3y^3$
20. $9xy^3,\ 72x^2y,\ 90xy^2,\ 108x^2y^2$

Objective 2 Factor out the greatest common factor.

Complete the factoring.

21. $48 = 8(\quad)$
22. $9x^3 = 3x(\quad)$
23. $-18y^8 = -3y^5(\quad)$
24. $-75a^4y^2 = 25a^2y(\quad)$

Factor out the greatest common factor.

25. $24r+9t$
26. $51m+34n$
27. $18q^2-45q$
28. $27xy+9x+18x^2$
29. $14w^2+84wx$
30. $24ab+8a+16ac$
31. $42tw+28t+70t^2$
32. $30rs+20r+50r^2$
33. $5a^4+25a^3-20a^2$
34. $13y^2-7$
35. $26x^8+13x^{12}-78x^{10}$
36. $100b^4+15b^2$
37. $12x^2y+48xy+30xy^2$
38. $5(a+b)+x(a+b)$
39. $c(a-b)-d(a-b)$
40. $x^2(r-4s)+y^2(r-4s)$

Objective 3 Factor by grouping.

Factor by grouping.

41. $y^2+3y+7y+21$
42. $a^2+7a+3a+21$
43. $d^2-5d+2d-10$
44. $y^2-7y+3y-21$
45. $2m^2+10m+2mn+5n$
46. $12z^2+18zq-2zq-3q^2$
47. $1-m+mn-n$
48. $4w+12-zw-3z$
49. $6y-xy+11x-66$
50. $6a^4+2a^3b^2-3ab^3-b^5$
51. $2a^3+2ab^2-3a^2b-3b^3$
52. $12r^3-3rs-4s^2r^2+s^3$
53. $2x^4+x^3y+4xy^3+2y^4$
54. $28-4d-7c+cd$

Mixed Exercises

Find the greatest common factor for the list.

55. 84, 28, 14

56. 81, 18, 126

57. $65y^4,\ 39y^7,\ 91y^4$

58. $68pq^3,\ 17p^2q^2,\ 34p^3q^2$

59. $35r^2s^2,\ 15r^3t^2,\ 21s^3t$

60. $-42u^2v^2,\ -105uv^2,\ -63uv^4$

Factor completely.

61. $21a^2 - 35a$

62. $7pq + 21q^2$

63. $5a^2b + 20ab + 15ab^2$

64. $2a(y - x) - 5b(y - x)$

65. $m^2 - 15m + 4m - 60$

66. $28 - 14m - 2m + m^2$

67. $10m^2 + 18m - 5m - 9$

68. $5r^3 + 5rs^2 - 4r^2s - 4s^3$

69. $9m^2n - 63mn + 9$

70. $1 + p - q - pq$

71. $44x^{12} - 33x^{10} + 22x^7$

72. $20y^6m^3 - 36y^2m^5$

73. $13y^3 - 39y^2$

74. $45q^4p^4 - 36q^2p^3 + 81q^2p^4$

75. $8 - 12p + 6p^3 - 9p^4$

76. $12c^2 + 18c - 10c - 15$

77. $6mn^2 - 42mn$

78. $96a^5 + 48a^3 + 4a^2$

79. $28m^2 + 20mn - 7m - 5n$

80. $4x^3 + 4xy^2 + 5x^2y + 5y^3$

Writing/Conceptual Exercises

81. Give an example of four numbers whose greatest common factor is 7.

82. Give an example of three terms whose greatest common factor is $3xy^5z^2$.

Determine whether the expression is *in factored form* or *not in factored form.*

83. $(3r-s)(3r-s)$

84. $2(3r-s)(s(3r-s))$

85. $x^2(y^2-z)-4(y^2-z)$

86. $(5-x^2)(9-y^2)$

Tell whether it is possible to factor the expression. If so, factor it. If not, explain why it cannot be factored.

87. $2x(3y-4)+5(3y+4)$

88. $c^2(2u+v)+d^2(2u+v)$

Section 6.2 Factoring Trinomials

Objective 1 Factor trinomials with a coefficient of 1 for the squared term.

List all pairs of integers with the given product. Then find the pair whose sum is given.

89. Product: 28; sum: –11

90. Product: 40; sum: –13

91. Product: –16; sum: 0

92. Product: –54; sum: –3

Complete the factoring.

93. $x^2+8x+15=(x+5)(\quad)$

94. $t^2-12t+35=(t-5)(\quad)$

95. $m^2+10m-24=(m-2)(\quad)$

96. $p^2-p-12=(p+3)(\quad)$

Factor completely. If the polynomial cannot be factored, write *prime*.

97. $y^2+10y+24$

98. $x^2-12x+27$

99. b^2-b-2

100. $z^2-10z+25$

101. $x^2-12x-45$

102. $n^2-10n-39$

103. $r^2+18r+17$

104. $x^2-15xy+56y^2$

105. $x^2-2xy-35y^2$

106. $m^2-2mn-3n^2$

Objective 2 Factor such trinomials after factoring out the greatest common factor.

Factor completely.

107. $3a^2 + 24a + 45$

108. $2p^3 + 8p^2 - 10p$

109. $3h^3k + 24h^2k - 27hk$

110. $8b^2 - 8b - 160$

111. $7a^2 - 35a + 42$

112. $2p^7 - 10p^6 - 28p^5$

113. $a^3b + a^2b^2 - 6ab^3$

114. $2x^2 - 30x + 100$

115. $8n^2 - 72n + 144$

116. $5h^2j - 30hj + 40j$

117. $10q^6 + 120q^5 + 320q^4$

118. $16x^7 - 48x^6 + 32x^5$

119. $6x^3y^2 + 30x^2y^3 + 36xy^4$

120. $5a^4b - 50a^3b^2 + 105a^2b^3$

Mixed Exercises

Factor completely. If the polynomial cannot be factored, write *prime*.

121. $a^2 - 10a + 16$

122. $w^2 - w + 5$

123. $x^3 - 7x^2 + 6x$

124. $5r^3 - 55r^2 + 150r$

125. $y^2 - 3y - 40$

126. $2m^3 + 6m^2 - 20m$

127. $2n^5 - 8n^4 + 6n^3$

128. $q^2 - 12q - 28$

129. $c^2 - 8cd + 16d^2$

130. $r^{10} - 4r^9q - 21r^8q^2$

131. $c^2 - 12cd + 35d^2$

132. $4m^2 - 8m + 4$

133. $2s^2t - 22st + 56t$

134. $2x^3 - 14x^2y + 20xy^2$

135. $2m^2 + 12m + 10$

136. $4n^2 + 20n + 16$

137. $x^2 - 5xy - 24y^2$

138. $p^2 + 13p + 36$

139. $4r^2 - 48r + 108$

140. $10z^2 + 20z - 350$

141. $5a^2 + 5ab - 10b^2$

142. $f^2 + fg - 30g^2$

143. $2s^2 - 40s + 198$

144. $-15y^3 - 2xy^2 + x^2y$

145. $2r^3 - 6r^2 - 20r$

146. $3a^3 + 15a^2 - 42a$

Writing/Conceptual Exercises

147. Which one of the following is the correct factored form of $z^2 + 7z - 18$?

(a) $(z-9)(z-2)$ (b) $(z+9)(z+2)$

(c) $(z+9)(z-2)$ (d) $(z-9)(z+2)$

148. Which one of the following is the correct factored form of $r^2 - 12r + 27$?

(a) $(r-3)(r-9)$ (b) $(r+3)(r+9)$

(c) $(r+3)(r-9)$ (d) $(r-3)(r+9)$

149. What would be your first step in factoring $3x^2 - 39x + 120$?

150. What would be your first step in factoring $4k^3 + 8k^2 - 48k$?

151. If you are asked to completely factor $4p^2 + 6p - 24,$ would it be correct to give $(p-2)(3p+12)$ as your answer? Explain.

152. Explain how you can be sure that $s^2 - 4s + 7$ is a prime polynomial.

Section 6.3 More on Factoring Trinomials

Objective 1 Factor trinomials by grouping when the coefficient of the squared term is not 1.

Factor each trinomial by grouping.

153. $8b^2 + 6b + 1$

154. $2x^2 + 15x + 28$

155. $15b^2 - 7b - 2$

156. $2m^2 + 7m + 6$

157. $9b^2 + 16b - 4$

158. $3m^2 + 20m + 12$

159. $7p^3 + 29p^2 + 4p$

160. $8m^2 - 26mn + 6n^2$

161. $7a^2b + 10ab - 8b$

162. $2s^2 + 19st - 10t^2$

163. $6y^2 - 3yz - 3z^2$

164. $15a^2 + 31ab + 10b^2$

Objective 2 Factor trinomials by using the FOIL method.

Complete the factoring.

165. $2a^2 + 9a - 5 = (2a - 1)(\quad)$

166. $16k^2 + 24k + 5 = (4k + 1)(\quad)$

167. $8t^2 + 14t - 15 = (2t + 5)(\quad)$

168. $24y^2 - 23y + 5 = (3y - 1)(\quad)$

Factor each trinomial by trial and error (using FOIL backwards).

169. $10x^2 + 9x + 2$

170. $4z^2 + 4z - 3$

171. $2a^2 - 13a - 7$

172. $15w^2 - 29w - 2$

173. $15q^2 + 11q + 2$

174. $14m^2 - 19m - 3$

175. $2m^2 + 2m - 12$

176. $15q^2 + 23q + 6$

177. $3a^2 - 5ab - 2b^2$

178. $6x^2 - 11xy - 10y^2$

179. $10s^2 - 9sw - 9w^2$

180. $6x^2 - 11xy - 10y^2$

181. $32x^2 - 28xy + 3y^2$

182. $16y^2 + 8y - 15$

Mixed Exercises

Factor completely.

183. $2a^2 + 7a + 3$

184. $6p^2 + 13p + 5$

185. $7y^2 + 6y - 1$

186. $8z^2 + 10z + 3$

187. $3p^2 - 14p - 5$

188. $9r^2 + 12r - 5$

189. $9x^2 - 26x - 3$

190. $6x^2 + 4xy - 10y^2$

191. $2x^4 - 11x^3 + 12x^2$

192. $12a^2 + 10a^2b - 12ab^2$

193. $18r^2 - 25rt + 8t^2$

194. $2y^4z^3 - 5y^3z^4 - 3y^2z^5$

195. $60x^4y - 65x^3y^2 - 175x^2y^3$

196. $108x^2y^5z^2 - 6x^2y^4z^3 - 80x^2y^3z^4$

197. $288x^7 - 68x^6y - 288x^5y^2$

198. $90x^2 + 15xyz - 50y^2z^2$

199. $63x^2 + 64xy - 55y^2$

200. $28 + 3z - z^2$

201. $25 - 25x + 6x^2$

202. $-16x^3 - 28x^2 + 30x$

203. $6y^3 - 19y^2 - 7y$

204. $-x^2 - x + 20$

205. $-8a^2 + 2ab + b^2$

206. $-6p^2 + 5pq + 4q^2$

Writing/Conceptual Exercises

207. Which one of the following is the correct factored form of $10x^2 - x - 2$?

(a) $(5x-2)(2x+1)$ (b) $(5x+2)(2x-1)$

(c) $(5x+1)(2x-2)$ (d) $(5x-1)(2x+2)$

208. Which one of the following is the correct factored form of $6r^2 - rs - 5s^2$?

(a) $(3r+5s)(2r-s)$ (b) $(3r-5s)(2r+s)$

(c) $(6r+5s)(r-s)$ (d) $(6r-s)(r+5s)$

209. For the trinomial $8x^2 - 11x + 3$, 4 is not a common factor. Explain how we know, then, that $4x - 8$ cannot be a factor of this polynomial.

210. If $6y^2 - 83y + 100$ is written in factored form, what will be the signs of the last terms of the two binomial factors?

Section 6.4 Special Factoring Techniques

Objective 1 Factor a difference of squares.

Factor the binomial completely. If it cannot be factored, write *prime*.

211. $t^2 - 121$

212. $x^2 + 36$

213. $64 - c^2$

214. $y^2 - 225$

215. $4a^2 - 49$

216. $9q^2 - \frac{16}{49}$

217. $144b^2 - 49$

218. $121m^2 - 36n^2$

219. $m^4 - 81$

220. $z^4 - 169$

221. $a^4 - 10,000$

222. $625y^4 - 1$

223. $9x^2 + 4$

224. $m^4 - n^2m^2$

Objective 2 Factor a perfect square trinomial.

Factor each trinomial completely. It may be necessary to factor out the greatest common factor first.

225. $y^2 - 4y + 4$

226. $q^2 + 18q + 81$

227. $t^2 - 12t + 36$

228. $c^2 - 22c + 121$

229. $z^2 - \frac{2}{3}z + \frac{1}{9}$

230. $9w^2 - 6w + 1$

231. $36q^2 - 36q + 9$

232. $49m^2 + 28m + 4$

233. $64p^4 + 48p^2q^2 + 9q^4$

234. $100p^2 - \frac{25}{2}pr + \frac{25}{64}r^2$

235. $r^2 + 2.8r + 1.96$

236. $-3x^2 + 6x - 3$

237. $-12a^2 - 60a - 75b^2$

238. $32x^2 - 48xy + 18y^2$

Objective 3 Factor a difference of cubes.

Factor completely.

239. $s^3 - 1$

240. $e^3 - 27$

241. $z^3 - 8$

242. $c^3 - 216$

243. $c^3 - 125$

244. $h^3 - 64$

245. $8w^3 - 1$

246. $8z^3 - 125$

247. $27r^3 - 8$

248. $343t^3 - 1$

249. $c^9 - d^6$

250. $r^3 - t^6$

251. $p^3 - 8q^3$

252. $125m^3 - 8p^3$

253. $8z^3 - 27y^3$

254. $27x^3 - 64y^3$

255. $8k^3 - \frac{1}{27}$

256. $b^3 - \frac{1}{8}$

257. $125c^3 - 343d^3$

258. $1000a^3 - 27b^3$

Objective 4 Factor a sum of cubes.

Factor completely.

259. $a^3 + 1$

260. $y^3 + 27$

261. $z^3 + 8$

262. $b^3 + 64$

263. $x^3 + 216$

264. $z^3 + 125$

265. $27q^3 + 1$

266. $8b^3 + 1$

267. $125c^3 + 8$

268. $343k^3 + 27$

269. $d^6 + 1$

270. $a^6 + 125b^3$

271. $w^6 + 8z^3$

272. $64x^3 + 27y^3$

273. $27s^3 + 8t^3$

274. $125g^6 + 1$

275. $216m^3 + 125p^3$

276. $j^6 + k^9$

277. $w^3 + \frac{1}{8}$

278. $27t^3 + \frac{1}{64}$

Mixed Exercises

Factor completely. If the polynomial cannot be factored, write *prime*.

279. $b^2 - 14b + 49$

280. $100k^2 - m^2$

281. $4p^2 - 121$

282. $4g^2 - 4g + 1$

283. $16r^2 + 49$

284. $25h^2 + 30h + 9$

285. $8x^3 + 125$

286. $8y^3 + 27z^3$

287. $a^4 - 14a^2b + 49b^2$

288. $k^2 - 1.2k + 0.36$

289. $3x^4 - 24x$

290. $c^4 - 625$

291. $e^2 + \frac{1}{3}e + \frac{1}{36}$

292. $\frac{1}{8}x^3 + 64$

293. $x^6 - \frac{1}{125}$

294. $\frac{1}{9}x^2 + 4xy + 36y^2$

295. $x^6 - 343y^3$

296. $16m^2 + 100$

297. $e^9 - 1$

298. $64b^2 - 64b + 16$

299. $-3x^2 - 30x - 75$

300. $x^8 - y^9$

301. $125x^3 + 64y^6$

302. $12z^3 - 12$

303. $121t^4 - 144v^6$

304. $81p^4 - 16q^4$

Writing/Conceptual Exercises

Identify the monomial as *a perfect square, a perfect cube, both of these,* or *neither of these*.

305. $81r^8$

306. $64q^6$

307. $81r^3$

308. $x^{12}y^6$

309. $121z^7$

310. $64u^{18}v^{24}$

311. Give an example of a sum of two squares that *can* be factored. Explain how to find other examples of this type of binomial.

312. A student factors x^2+36 as $(x+6)(x+6)$ or $(x+6)^2$. Explain to the student why this is incorrect.

Section 6.5 Solving Quadratic Equations by Factoring

Objective 1 Solve quadratic equations by factoring.

Solve the equation and check the answers.

313. $(y+6)(2y-5)=0$

314. $(2c-5)(3c+4)=0$

315. $x^2+2x-8=0$

316. $b^2-81=0$

317. $z^2+11z+24=0$

318. $w^2+4w=32$

319. $6r^2=48r$

320. $2y^2+15=11y$

321. $5y^2+3y=2$

322. $3x^2+5x=28$

323. $25x^2+4=20x$

324. $z^2-121=0$

325. $6p^2+7p=20$

326. $15x^2-24x=12$

327. $c(5c+17)=12$

328. $4x(x+2)=(x-1)^2-1$

Objective 2 Solve other equations by factoring.

Solve the equation.

329. $3x(x+9)(x+2)=0$

330. $x(3x^2+14x-5)=0$

331. $z(4z^2-81)=0$

332. $z^3-36z=0$

333. $f^3=144f$

334. $y^3-2y^2-3y=0$

335. $3m^3+2m^2-8m=0$

336. $(p-2)(p^2-49)=0$

337. $z^4+9z^3-10z^2=0$

338. $5z^3+14z^2=3z$

339. $(x+2)(x^2-10x+21)=0$

340. $(y^2-5y+6)(y^2-25)=0$

341. $x^3=-x^2+20x$

342. $(y-3)(2y^2+3y-2)=0$

343. $\left(x-\frac{1}{2}\right)(2x^2-x-15)=0$

344. $\left(y-\frac{2}{3}\right)\left(y^2-\frac{1}{25}\right)=0$

Mixed Exercises

Solve the equation.

345. $(x+7)(2x+9)=0$

346. $x(x+5)(x-7)=0$

347. $x^2+7x+10=0$

348. $x^2=7x$

349. $2x(x^2+2x-15)=0$

350. $5x^2=38x+16$

351. $9p^2-25=0$

352. $k^3=169k$

353. $(x-3)(x^2-36)=0$

354. $3y^2+7y=40$

355. $v^5-6v^4+8v^3=0$

356. $b(3b+11)=4$

357. $x^2 = 14x - 45$

358. $(z-3)(z^3 - z) = 0$

359. $2x^2 - 7x = 15$

360. $16x^2 = 0$

361. $x^2 - 7x = 44$

362. $2(x+6)(x-5) = 0$

363. $x^3 = 36x$

364. $4x^2 - 81 = 0$

365. $b^2(b+4)(b-9) = 0$

366. $5x^2 - 85x + 360 = 0$

367. $(x-6)(x+3)(x-5) = 0$

368. $(x^2 - 4)(x^2 - 16) = 0$

369. $x^2(x^2 - 81)(x^2 - 1) = 0$

370. $8x\left(x - \frac{1}{5}\right)\left(x - \frac{1}{2}\right) = 0$

Writing/Conceptual Exercises

Write a quadratic in standard form having the given solutions. Use x as the variable.

371. 2 and –9

372. 6 and $\frac{3}{2}$

373. $-\frac{1}{2}$ and $-\frac{3}{5}$

374. $\frac{5}{4}$ and $-\frac{3}{2}$

375. 0 and –9

376. –6 (only solution)

377. Explain why the solutions of the equation $(x-1)(x+1) = 6$ are not found by solving the equations

$$x - 1 = 6 \text{ and } x + 1 = 6.$$

378. What is wrong with the following solution?

$$10x^2 = 2x$$
$$5x = 1$$
$$x = \frac{1}{5}$$

Show the correct way to solve this equation.

Section 6.6 Applications of Quadratic Equations

Objective 1 Solve problems involving geometric figures.

Solve the problem.

379. Then length of a book is 4 centimeters more than its width. The area is 77 square centimeters. Find the length and width of the book.

380. The length of a rectangle is twice its width. If the width were increased by 3 and the length remained the same, the resulting rectangle would have an area of 140 square inches. Find the dimensions of the original rectangle.

381. The area of a piece of metal sheeting is 135 square feet. Its width is 6 feet less than its length. Find the length and width of the sheeting.

382. Two rectangles with different dimensions have the same area. The length of the first rectangle is four times its width. The length of the second rectangle is 12 meters more than the width of the first rectangle, and its width is 2 meters more than the width of the first rectangle. Find the lengths and widths of the two rectangles.

383. Each side of one square is 1 meter less than twice the length of each side of a second square. If the difference between the areas of the two squares is 16 square meters, find the lengths of the sides of the two squares.

384. The area of a triangular sticker is 56 square centimeters. The base is 2 centimeters less than twice the height. Find the base and height of the sticker.

385. A rectangular circuit board is 5 centimeters longer than it is wide. Its area is numerically 2 more than its perimeter. Find the length and width of the circuit board.

386. A calculator is twice as long as it is wide. Find the length and width of the calculator in centimeters if its area is numerically 56 more than its perimeter.

387. The volume of a box is 352 cubic feet. If the width of the box is 8 feet and the length is 7 feet more than the height, find the height of the box.

388. A carpenter is building a box which will have a volume of 60 cubic meters. The height of the box will be 4 meters, and the length will be 2 meters more than the width. Find the width of the box.

Objective 2 Solve problems involving consecutive integers.

389. Find two consecutive integers such that the square of their sum is 49.

390. The product of two consecutive integers is 4 less than twice their sum. Find the integers.

391. Find all possible pairs of consecutive odd integers whose sum is equal to their product decreased by 7.

392. The square of the sum of two consecutive even integers is 100 more than twice their product. Find the integers.

393. The product of two consecutive even integers is 32 more than twice the larger. Find the integers.

394. Find three consecutive odd integers such that four times the sum of all three equals 13 more than the product of the smaller two.

Objective 3 Solve problems by using the Pythagorean formula.

Solve the problem.

395. The hypotenuse of a right triangle is 9 inches longer than the shorter leg. The longer leg is 7 inches longer than the shorter leg. Find the length of the shorter leg.

396. The hypotenuse of a right triangle is 2 meters shorter than three times the shorter leg, and the longer leg is 2 meters longer than twice the shorter leg. Find the length of the hypotenuse.

397. A lot has the shape of a right triangle with one leg 4 meters longer than twice the length of the other leg. The hypotenuse is 4 meters less than three times the length of the shorter leg. Find the dimensions of the lot.

398. A train and a car leave a station at the same time, the train traveling due north and the car traveling due west. When they are 100 miles apart, the train has traveled 20 miles farther than the car. Find the distance each has traveled.

399. A ladder is leaning against the side of a building so that the distance from the bottom of the ladder to the building is 4 feet less than the distance up the building to the top of the ladder. If the ladder is 4 feet shorter than twice the distance from the bottom of the ladder to the building, how long is the ladder?

400. Rhea is standing directly beneath a kite attached to a string which Steve is holding with his hand touching the ground. The height of the kite at that instant is 5 feet more than twice the distance between Rhea and Steve. The length of the kite string is 23 feet less than three times the distance. Find the length of the kite string.

401. A 15-foot ladder is leaning against a building. The distance from the bottom of the ladder to the building is 3 feet less than the distance from the top of the ladder to the ground. How far is the bottom of the ladder from the building?

402. A lot is in the shape of a right triangle. The shorter leg measures 60 meters. The hypotenuse measures 18 meters more than the longer leg. Find the dimensions of the lot.

403. Two cars left an intersection at the same time. One traveled south. The other traveled 9 miles farther, but to the east. How far apart were they when the distance between them was 9 miles more than the distance traveled east?

404. A ladder is leaning against a building. The distance from the bottom of the ladder to the building is 8 feet less than the length of the ladder. How high up the side of the building is the top of the ladder if that distance is 4 feet less than the length of the ladder?

Objective 4 Solve problems by using given quadratic models.

Use the quadratic model to answer the questions.

The equation $y = -.02x^2 + 1.19x + 27$ was developed to model fuel economy trends within the automobile industry starting in 1999. Suppose that an automotive engineer is revising the model to project fuel economy trends into the 21st century. She develops the following formula:

$$y = -.02x^2 + 1.35x + 40,$$

and determines that x is coded so that $x = 0$ represents 1999.

405. Calculate the expected miles per gallon in 2009. Round your answer to the nearest tenth.

406. Calculate the expected miles per gallon in 2029. Round your answer to the nearest tenth.

407. Calculate the expected miles per gallon in 2099. Round your answer to the nearest tenth.

408. Would you buy a car in 2099 based on the answer question #407? Why or why not?

Mixed Exercises

Solve the problem.

409. The length of a rectangular picture is 5 centimeters more than the width. The area is 84 square centimeters. Find the length and width of the picture.

410. The length of a rectangular card is twice its width. If the area were increased by 2 inches while the length remained the same, the resulting rectangle would have an area of 48 square inches. Find the dimensions of the original card.

411. The length of the shorter leg of a right triangle is tripled and 1 inch is added to the result, giving the length of the hypotenuse. The longer leg is 11 inches longer than twice the shorter leg. Find the length of the shorter leg of the triangle.

412. Gerald wishes to build a box to hold his tools. The box is to be 4 feet high, and the width of the box is to be 1 foot less than the length. The volume of the box will be 120 cubic feet. Find the length and width of the box.

413. The hypotenuse of a right triangle is 1 foot longer than twice the shorter leg. The longer leg is 1 foot shorter than twice the shorter leg. Find the length of the shorter leg of the triangle.

414. Lori and Debbie started walking from the same corner, with Lori walking west and Debbie walking north. When they were 13 kilometers apart, Debbie had walked 7 kilometers farther than Lori. Find the distance each of them had walked.

415. The length of a rectangular label is three times its width. If the length were decreased by 1 while the width stayed the same, the area of the new label would be 70 square centimeters. Find the length and width of the original label.

416. The length of the floor of a rectangular closet is 1 foot more than the width. The area of the floor is 56 square feet. Find the length and width of the floor.

417. A bicyclist heading east and a motorcyclist traveling south left an intersection at the same time. When the motorcyclist had gone 14 miles farther than the bicyclist, the distance between them was 2 miles more than the distance traveled by the motorcyclist. How far apart were they then?

418. The sides of one square have a length of 2 meters more than the sides of another square. If the area of the larger square is subtracted from three times the area of the smaller square, the answer is 66 square meters. Find the lengths of the sides of each square.

419. If a ball is thrown upward from ground level with an initial velocity of 80 feet per second, its height h (in feet) t seconds later is given by the equation

$$h = -16t^2 + 80t.$$

(a) After how many seconds is the height 100 feet?

(b) After how many seconds will be ball hit the ground?

420. An object is propelled upward from a height of 22 feet with an initial velocity of 48 feet per second, its height h (in feet) t seconds later is given by the equation

$$h = -16t^2 + 48t + 22.$$

(a) After how many seconds is the height 58 feet?

(b) After how many seconds is the height 54 feet?